
Model Independent Analysis of the Solar Neutrino Data ^{*}

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Abstract

We perform an updated model-independent analysis using all the latest solar neutrino data, including the one coming from remarkably high statistics SuperKamiokande experiment. We confirm that the astrophysical solutions to the solar neutrino problem are extremely disfavored. We also present a new way of illuminating the suppression pattern of various solar neutrino flux, which indicates that the strong suppression of ^7Be neutrinos is no more true once the neutrino flavor conversion is taken into account.

1. Introduction

The solar neutrino problem [2] is now established essentially independent of any details of the standard solar models (SSM). The so called model-independent analysis was performed by several authors [3,10,4,19,16,24,5,17,13] which revealed that the ^7Be neutrinos must be strongly depleted compared to the SSM prediction. From these analyses one can conclude that the solar neutrino problem cannot be explained by astrophysical mechanisms unless some assumptions in the standard electroweak theory and/or solar neutrino experiments are grossly incorrect.

In this work, we repeat the model-independent analysis using the latest solar neutrino data as well as the new updated theoretical prediction by Bahcall

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and Pinsonneault (BP98) [7]. Our analysis will indicate that astrophysical solution such as the low- T model [2] is convincingly excluded by the present data. We also present a new way of illuminating the suppression pattern of various solar neutrino flux originated from different fusion reactions in a less model-dependent fashion. We will observe that the statement of the missing ${}^7\text{Be}$ neutrinos is no more true in the presense of neutrino flavor conversion.

Table 1. Observed solar neutrino event rates used in this analysis and corresponding theoretical predictions [7]. The quoted errors are at 1σ .

Experiment	Data $\pm(\text{stat.}) \pm(\text{syst.})$	Ref.	Theory [7]	Units
Homestake	$2.56 \pm 0.16 \pm 0.15$	[11]	$7.7^{+1.2}_{-1.0}$	SNU
SAGE	$69.9^{+8.0+3.9}_{-7.7-4.1}$	[14]	129^{+8}_{-6}	SNU
GALLEX	$76.4 \pm 6.3^{+4.5}_{-4.9}$	[18]	129^{+8}_{-6}	SNU
SuperKam	$2.44 \pm 0.05^{+0.09}_{-0.07}$	[23]	$5.15^{+0.98}_{-0.72}$	$10^6 \text{ cm}^{-2}\text{s}^{-1}$

2. The Data

The latest solar neutrino data, including the ${}^8\text{B}$ neutrino flux measured during 504 days in the SuperKamiokande experiment [23], which will be used in our analysis are tabulated in Table 1. From these data, by adding the statistical and the systematic errors quadratically, we obtain,

$$S_{Cl}^{obs} = 2.56 \pm 0.23 \text{ SNU}, \quad (1)$$

$$S_{Ga}^{obs} = 72.4 \pm 6.6 \text{ SNU}, \quad (2)$$

$$S_{SK}^{obs} = (2.44 \pm 0.10) \times 10^6 \text{ cm}^{-2}\text{s}, \quad (3)$$

where, to be conservative, we always take the larger values of statistical and systematic errors, whenever errors are asymmetric, in each experiment before we combine.

Our analysis in the present work is based only upon the total rate of each experiment, and the information of the energy spectrum of ${}^8\text{B}$ neutrinos obtained by SuperKamiokande experiment, is not taken into account. Thus, it is to illuminate the global features of the suppression of the solar neutrino spectrum.

3. Model-Independent Analysis

The fundamental assumptions in our analyses are as follows: (i) The sun shines due to the nuclear fusion reactions from which and only from which the solar neutrinos come, (ii) the relevant reactions responsible for creating neutrinos

in the sun are assumed to be those postulated in the SSM, (iii) the sun is quasi-stable during the time scale of 0.1-1 million years.

These assumptions (i) to (iii) imply that the solar neutrino flux generated by various nuclear fusion reactions must obey the luminosity constraint [21,22,15,24,5],

$$\frac{L_{\odot}}{4\pi R^2} = \sum_{\alpha} \left(\frac{Q}{2} - \langle E \rangle_{\alpha} \right) \Phi(\alpha) \quad (4)$$

where $L_{\odot} = 3.844 \times 10^{33}$ (erg/s), $R = 1$ A.U. (1.469×10^{13} cm), $Q = 26.73$ MeV is the energy release when one ${}^4\text{He}$ is created, $\langle E \rangle_{\alpha}$ and $\Phi(\alpha)$, ($\alpha = pp, {}^7\text{Be}, {}^8\text{B}, \dots$), are the average energy loss by neutrinos and the neutrino flux, respectively.

By normalizing the neutrino flux to those of the SSM of BP98 [7] as,

$$\phi^{\alpha} = \frac{\Phi(\alpha)}{\Phi(\alpha)_{SSM}}, \quad (\alpha = pp, {}^7\text{Be}, {}^8\text{B}, \dots), \quad (5)$$

the luminosity constraint is simply given by,

$$1 = 0.907\phi^{pp} + 0.0755\phi^{7Be} + 4.97 \times 10^{-5}\phi^{8B}, \quad (6)$$

where we have neglected the contribution from CNO and *pep* neutrinos.

In this section we make the following two more specific assumptions in addition to the fundamental assumptions (i) - (iii): (1) The energy spectra of the solar neutrinos are not modulated, (2) neutrino flavor transformation does not occur after neutrinos are created in the sun until they reach the detectors. Then the luminosity constraint is effective, and the flux Φ is the actual flux to be detected by the terrestrial detectors.

The expected solar neutrino signal to the ${}^{37}\text{Cl}$, ${}^{71}\text{Ga}$ and SuperKamiokande solar neutrino experiments are given in terms of neutrino flux by,

$$S_{Cl}^{th} = 5.9\phi^{8B} + 1.15\phi^{7Be} \quad \text{SNU}, \quad (7)$$

$$S_{Ga}^{th} = 12.4\phi^{8B} + 34.4\phi^{7Be} + 69.6\phi^{pp} \quad \text{SNU}, \quad (8)$$

$$R_{SK}^{th} = \phi^{8B}, \quad (9)$$

where we have neglected the contribution from the *pep* and the CNO neutrinos because the inclusion of them does not affect our conclusion in this section.

Using Eqs. (7-8) as well as the observed solar neutrino data summarized in Table I we perform a simple χ^2 analysis. After eliminating ϕ^{pp} from (8) by using the luminosity constraint we freely vary the two flux, ϕ^{8B} and ϕ^{7Be} and compute the χ^2 . The minimum χ^2 is reached when ϕ^{7Be} takes a negative value but we impose the condition $\phi^{7Be} \geq 0$ to be physically meaningful.

We plot in Fig. 1 the contours of χ^2 corresponding to 1σ , 2σ , ... 5σ , for two free parameters. We also plot in Fig. 1 the curve, $\phi^{7Be} = (\phi^{8B})^{10/24}$ which

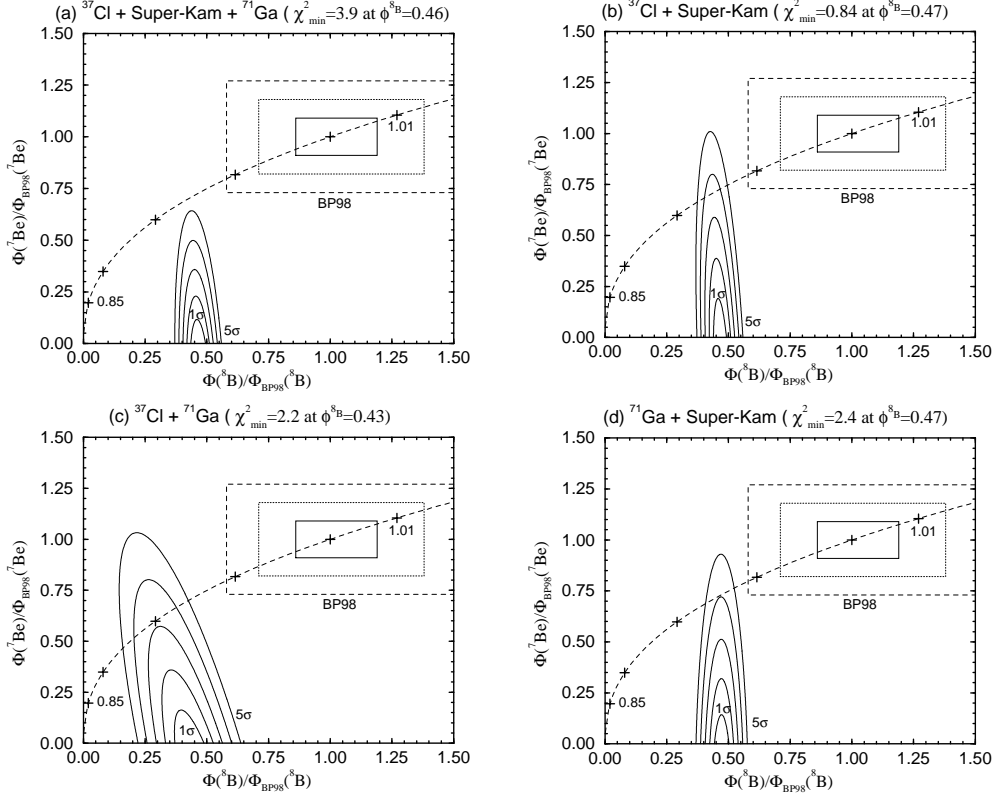


Fig. 1. Contour plot of the χ^2 values in the $\Phi_{sB} - \Phi_{7Be}$ plane for different combinations of the solar neutrino experiments. The solid curves correspond to 1 σ to 5 σ , with step size 1, from inside to outside. We also indicate the 1, 2 and 3 σ theoretical range predicted by BP98, by the solid, dotted and dashed lines, respectively. Along the dashed curve, $\phi^{7Be} = (\phi^8B)^{10/24}$, the crosses indicate, from left to right, the point where the central temperatures are 0.85, 0.9, 0.95, 0.98, 1 and 1.01 with respect to the prediction by the SSM.

corresponds to the case where the central temperature of the sun T_c is varied freely, but keeping the relationship between the two fluxes as [6], $\phi^{7Be} \propto T_c^{10}$ and $\phi^{8B} \propto T_c^{24}$.

As in the previous analyses [15,10,4,19,16,24,17] χ^2_{\min} is achieved at vanishing ^7Be flux not only in Fig. 1 (a) where all the experiments are taken into account but also in Fig. 1 (b-d) where only two experiments are included. By comparing Fig. 1, for example, with those of [15] and by [24], we can clearly see that width of the contours has been greatly shrunk along the ϕ^8B axis, indicating how significantly the result is affected by the high statistics data from SuperKamiokande.

From Fig. 1 we see that solar neutrino data are in strong disagreement with the SSM prediction as also concluded in ref. [8]. We conclude from Fig. 1 that the

standard solar model BP98 is ruled out by the current solar neutrino data at the significance level much higher than 5σ under our fundamental assumptions (i)-(iii) and the additional ones (1)-(2). We have also confirmed that the astrophysical solution of the solar neutrino problem, such as low- T model is strongly disfavored by the data. We stress that our basic conclusions do not change even if we neglect one of the three types of experiments.

4. Suppression Pattern of Neutrino Flux implied by the current Solar Neutrino Data

In this section we describe a new way of illustrating the suppression pattern of most relevant neutrino flux, required to explain the current solar neutrino experiments. We do this by taking into account the possibility of occurrence of either the active conversion, $\nu_e \rightarrow \nu_{\mu,\tau}$ or the sterile one $\nu_e \rightarrow \nu_s$ in between the solar core and the terrestrial detectors.

To obtain global understanding of the suppression pattern we propose to combine the *pep* and CNO neutrinos into the ${}^7\text{Be}$ neutrinos and denote them as the intermediate energy neutrinos. While it is more reasonable, in the context of model-independent analysis, to combine the *pep* neutrinos with the *pp*'s because they are competing partners in the *pp* I chain, here, we combine the flux when their energy regions overlap.

We assume, in this section (except in Subsec. 4.3), that neutrino production rates from each source are the same as the ones predicted by the BP98 SSM. Then the expected signal in each experiment in the presence of neutrino conversion, $\nu_e \rightarrow \nu_{\mu,\tau}$, is given by,

$$S_{Cl}^{th} = 5.9\langle P_B \rangle + 1.83\langle P_I \rangle \quad \text{SNU}, \quad (10)$$

$$S_{Ga}^{th} = 12.4\langle P_B \rangle + 46.9\langle P_I \rangle + 69.6\langle P_{pp} \rangle \quad \text{SNU}, \quad (11)$$

$$R_{SK}^{th} = \langle P_B \rangle + r(1 - \langle P_B \rangle), \quad (12)$$

where $\langle P_B \rangle$, $\langle P_I \rangle$ and $\langle P_{pp} \rangle$ are the average survival probabilities for ${}^8\text{B}$, intermediate energy and *pp* neutrinos, respectively. They are regarded as the average over the neutrino flux times the cross section, and as well as the detection efficiency in the case of the SuperKamiokande experiment. In eq. (12) r is essentially given by the ratio of the scattering cross section of $\nu_{\mu(\tau)}$ to that of ν_e off electron, i.e., $r \equiv \langle \sigma_{\nu_{\mu}e} \rangle / \langle \sigma_{\nu_e e} \rangle \simeq 0.16$. When we consider the case (in Subsec. 4.2) where the ${}^8\text{B}$ ν_e 's are converted into some sterile states, we set $r = 0$ in eq. (12).

In eqs. (12-14) we simply assume that the average survival probability for all the intermediate energy *pep*, CNO and ${}^7\text{Be}$ neutrinos are the same and denote it as $\langle P_I \rangle$ so that the coefficient of $\langle P_I \rangle$ in eqs. (12) and (13) now includes the

contribution not only from ${}^7\text{Be}$ but also from pep and CNO neutrinos (cf. eqs. (7) and (8)). Furthermore, we take, as an approximation, $\langle P_i \rangle$ ($i = pp, I, {}^8\text{B}$) to be equal for all the experiments because the energy dependences of the flux times cross section (times the detection efficiency for the SuperKamiokande) are rather similar among different experiments, as first noticed by Kwong and Rosen [19].

Other than these assumptions, we do not consider any specific mechanism of neutrino flavor transformation in this analysis but aim at illuminating global features of the modification of the solar neutrino spectrum. We try to determine the reduction rates of the flux of low, intermediate, and high energy neutrinos at the earth in such a way that the experimental data can be fitted.

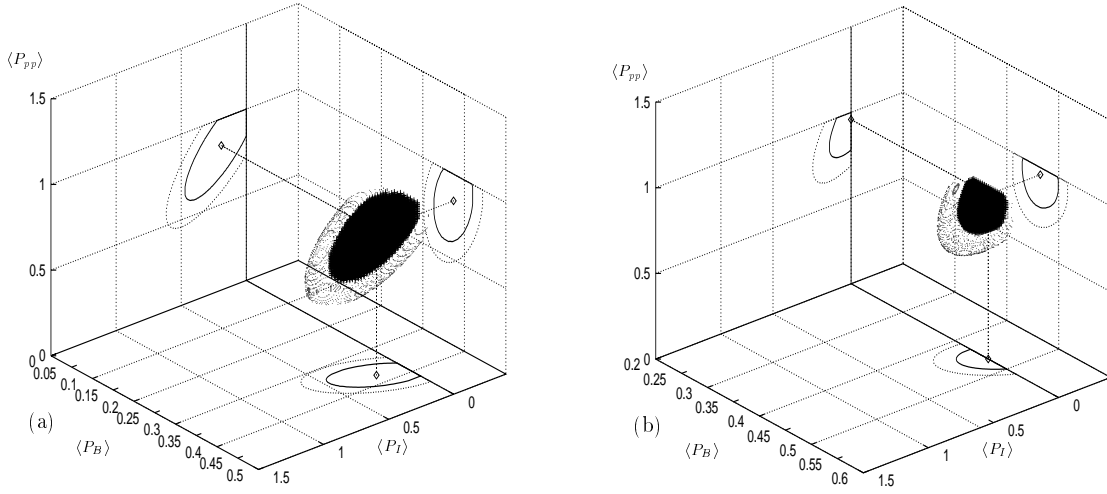


Fig. 2. Allowed range of neutrino flux determined by all the solar neutrino experiments with the condition $\chi^2 = \chi_{min}^2 + 3.5$ (1σ) and 8.0 (2σ) (for three free parameters) assuming the neutrino conversion $\nu_e \rightarrow \nu_{\mu,\tau}$ or ν_s . We show in (a) the active and in (b) the sterile conversions cases. The allowed ranges are projected into each plane, indicated by the solid curves (1σ) and the dotted curves (2σ). The best fitted reduction rates of the neutrino fluxes are, $(\langle P_B \rangle, \langle P_I \rangle, \langle P_{pp} \rangle) = (0.37, 0.19, 0.84)$ for active conversion with $\chi^2 \approx 0$, and $= (0.47, 0, 0.96)$ for sterile conversion with $\chi^2 \approx 0.8$.

4.1. The case of active neutrinos

We present our results in Fig. 2 (a) for the case of active conversion. We note that $\langle P_B \rangle$ is determined most accurately, as expected from the large statistics of the SuperKamiokande experiment. On the other hands, the other two, $\langle P_I \rangle$ and $\langle P_{pp} \rangle$, have larger uncertainties at the present stage of the solar neutrino data. We stress that the proposed experiments such as Borexino [1], Hellaz [20] and Heron

[9] are needed in order to determine the ${}^7\text{Be}$ and pp neutrino flux more accurately, especially if the conversion mechanism is unknown. We also tabulate the range of allowed values of the survival probabilities with their 1σ uncertainties in Table 2.

From Fig. 2 and Table 2 we can see that strong suppression of intermediate energy neutrinos, the one best fit by negative flux, is no more true when the neutrino flavor conversion is taken into account. This feature is in sharp contrast with the results of the model-independent analysis in Sec. 3 and of the flavor conversion into sterile neutrinos to be discussed below.

We also note that the suppression rate of the intermediate-energy neutrinos depends rather sensitively on the presense or absence of the pep and CNO neutrinos. If we ignore their contribution the best fit value of $\langle P_I \rangle$ ($=\langle P_{Be} \rangle$) becomes larger by a factor of 2 (see Table 2).

Table 2. The range of reduction rates of each neutrino flux with respect to the prediction by BP98 SSM implied by the solar neutrino data. We present the cases with (a) and without (b) pep and CNO contribution.

Case	$\langle P_B \rangle$	$\langle P_I \rangle$	$\langle P_{pp} \rangle$
Active (a)	0.33 – 0.42	0 – 0.46	0.6 – 1
Active (b)	0.33 – 0.42	0 – 0.74	0.55 – 1
Sterile (a)	0.43 – 0.50	0 – 0.16	0.77 – 1
Sterile (b)	0.43 – 0.50	0 – 0.26	0.76 – 1

4.2. The case of sterile neutrinos

We next consider the case where the neutrinos are converted into sterile species. Since only the water-Cherenkov experiment can be sensitive to the difference between conversions into active and sterile neutrinos any change in our result from the active case solely comes from ${}^8\text{B}$ neutrinos. The results for the sterile neutrino conversion is presented in Fig. 2 (b) and in Table 2. By comparing Fig. 2(a) and (b), we can clearly see that the stronger suppression of ${}^7\text{Be}$ neutrinos is required than the case of active conversion in the case with $\phi^{8B} = 1$. We note that the best fit is obtained when the flux of intermediate energy neutrino is negative.

4.3. Varying ${}^8\text{B}$ flux

Finally, we discuss the sensitivity of our results against the change in the neutrino flux from those of the SSM. Since the pp neutrino flux is essentially fixed by the solar luminosity and also the ${}^7\text{Be}$ neutrino flux are better determined

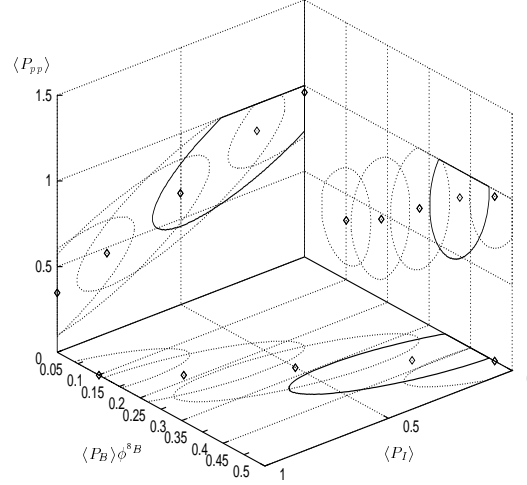


Fig. 3. Two sigma allowed range of neutrino flux, projected into each plane, assuming the neutrino conversion $\nu_e \rightarrow \nu_\mu$ or ν_τ , for different values of ϕ^{8B} are plotted by the dotted curves (except for the $\phi^{8B} = 1$ case). The five curves in the each plane correspond, from left to right, to the case where $\phi^{8B} = 2.5, 2.0, 1.5, 1.0$ and 0.5 . The corresponding best fitted reduction rates, indicated by open diamonds are, $(\langle P_B \rangle \phi^{8B}, \langle P_I \rangle, \langle P_{pp} \rangle) = (0.1, 1.0, 0.35), (0.18, 0.81, 0.46), (0.28, 0.50, 0.65), (0.37, 0.19, 0.84)$ and $(0.46, 0.0, 0.96)$.

compared to the ^8B flux which is subject to the uncertainty of the nuclear cross section S_{17} , we only vary the ^8B flux and examine the sensitivity of the required reduction rates against its change.

We will perform this exercise only for the active neutrino conversion case since for the sterile case the result presented in Fig. 2 (b) still holds if $\langle P_B \rangle$ is regarded as $\langle P_B \rangle \phi^{8B}$ if we vary ϕ^{8B} , whereas for the active case, this is not true, because in the water-Cherenkov experiment, the event rate depends not only on $\phi^{8B} \langle P_B \rangle$ but also on ϕ^{8B} itself.

In Fig. 3 we plot the allowed range of the reduction rates of the neutrino flux by (artificially) varying the ^8B neutrino flux prediction of the SSM. The result of the exercise indicates that as the ϕ^{8B} gets larger, preferred value of $\langle P_I \rangle$ becomes larger, while $\langle P_{pp} \rangle$ gets smaller as seen in the plot. This feature is consistent with one obtained by Smirnov in Table I of [25].

Let us note that the arbitrariness of the interpretation of which ϕ^{8B} or $\langle P_B \rangle$ are changed from the standard theory can be removed if we combine the results of the SuperKamiokande and either one of charged or (preferable) neutral current data from the SNO experiment [12]. One can separately estimate the flux of ^8B neutrinos and the survival probability by combining these two experiments.

5. Conclusions

We have performed the updated model-independent analysis of the current solar neutrino data. We confirmed, with current data of any two sets out of the three, the ^{37}Cl , the ^{71}Ga and the SuperKamiokande experiments that: (1) the SSM prediction can be convincingly rejected, and (2) the ^7Be neutrinos is strongly suppressed unless ^8B neutrinos are converted into another active flavor. We have shown that the low- T model is excluded by more than 5σ (3σ) with data of the three (two out of the three) experiments. The best fitted value of ^7Be neutrino flux is always negative even if we do not impose the luminosity constraint.

On the other hand, if we assume that neutrino flavor conversion of $\nu_e \rightarrow \nu_\mu$ or ν_τ is occurring, the best fitted flux of ^7Be (or intermediate energy) neutrino is no longer negative. The current solar neutrino data suggest, as the best fit in our analysis, that $(\langle P_B \rangle, \langle P_I \rangle, \langle P_{pp} \rangle) \sim (0.4, 0.2, 0.8)$. While it is still suppressed the value of the intermediate energy neutrinos makes most notable difference between cases with and without neutrino flavor conversion. We hope that this point is resolved by the future solar neutrino experiments.

More detailed discussions and relevant references are found in ref. [21].

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